HOSSAM GHANEM

(38) 4.10 Antiderivatives, Indefinite Integrals, And Simple Differential Equations

$\int x^n\,dx = \frac{1}{n+1}x + C$	$\int \sin x\;dx = -\cos x + C$
$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + C$	$\int \cos x\;dx = \sin x + C$
$\sin^2 x = \frac{1}{2}(1-\cos\,2x)$	$\int \sec^2 x\;dx = \tan x + C$
$\cos^2 x = \frac{1}{2}(1+\cos\,2x)$	$\int \csc^2 x\;dx = -\cot\,x + C$
$\tan^2 x = \sec^2 x\,-1$	$\int \sec x\tan x\;dx = \sec x + C$
$\cot^2 x = \csc^2 x\,-1$	$\int \csc x\cot x\;dx = -\csc x + C$

Example 1	$\int (5x^3 - 2x^2 + 3x - 7)dx$

Solution

$$\int (5x^3 - 2x^2 + 3x - 7)dx = \frac{5}{4}x^4 - \frac{2}{3}x^3 + \frac{3}{2}x^2 - 7x + c$$

Example 2	$\int \left(\frac{3}{x^7} - \frac{5}{x^2} + 2x\right)dx$

Solution

$$I = \int \left(\frac{3}{x^7} - \frac{5}{x^2} + 2x\right)dx = \int (3x^{-7} - 5x^{-2} + 2x)\,dx$$

$$= \int \frac{-3}{6}x^{-6} + 5x^{-1} + x^2 + c \quad = \frac{-1}{2x^6} + \frac{5}{x} + x^2 + c$$

Example 3	$\int \left(\sqrt{u^5} - \frac{1}{3}u^{-2} + 6\right)du$

Solution

$$I = \int \left(\sqrt{u^5} - \frac{1}{3}u^{-2} + 6\right)du = \int \left(u^{\frac{5}{2}} - \frac{1}{3}u^{-2} + 6\right)du = \frac{2}{7}u^{\frac{7}{2}} + \frac{1}{3}u^{-1} + 6u + c$$

Example 4	$\int \frac{2x^2 - x + 3}{\sqrt{x}}\,dx$

Solution

$$I = \int \frac{2x^2 - x + 3}{\sqrt{x}}\,dx = \int \frac{2x^2 - x + 3}{x^{\frac{1}{2}}}\,dx = \int \left(2x^{\frac{3}{2}} - x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}\right)dx$$

$$= 2\cdot\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + 3\cdot 2x^{\frac{1}{2}} + c = \frac{4}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + 6x^{\frac{1}{2}} + c$$

Example 5	$\int \frac{x^3 - 3}{x^2}\,dx$

Solution

$$I = \int \frac{x^3 - 3}{x^2}\,dx = \int (x - 3x^{-2})dx = \frac{1}{2}x^2 + 3x^{-1} + c = \frac{1}{2}x^2 + \frac{3}{x} + c$$

Example 6	$\int \left(x^2 - \frac{1}{x^2}\right)^2 dx$

Solution

$$I = \int \left(x^2 - \frac{1}{x^2}\right)^2 dx = \int \left(x^4 - 2 + \frac{1}{x^4}\right)dx = \int (x^4 - 2 + x^{-4})dx = \frac{1}{5}x^5 - 2x - \frac{1}{3}x^{-3} + c$$

$$= \frac{1}{5}x^5 - 2x - \frac{1}{3x^3} + c$$

Example 7	$\int (3x-2)(2x^2+1)dx$

Solution

$$I = \int (3x-2)(2x^2+1)dx = \int (6x^3 + 3x - 4x^2 - 2)dx = \frac{6}{4}x^4 + \frac{3}{2}x^2 - \frac{4}{3}x^3 - 2x + c$$

$$= \frac{6}{4}x^4 - \frac{4}{3}x^3 + \frac{3}{2}x^2 - 2x + c$$

Example 8	$\int \sqrt{x}\left(\sqrt{x} - \sqrt{3}\right)dx$

Solution

$$I = \int \sqrt{x}\left(\sqrt{x} - \sqrt{3}\right)dx = \int \left(x - \sqrt{3}\,x^{\frac{1}{2}}\right)dx = \frac{1}{2}x^2 - \sqrt{3}\cdot\frac{2}{3}x^{\frac{3}{2}} + c = \frac{1}{2}x^2 - \frac{2}{\sqrt{3}}x^{\frac{3}{2}} + c$$

Example 9	$\int \frac{x^3 - 2x^2 - 5x + 6}{x - 3} dx$

Solution

$$I = \int \frac{x^3 - 2x^2 - 5x + 6}{x - 3} dx = \int (x^2 + x - 2) dx$$
$$= \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + c$$

$$\begin{array}{r} x^2 + x - 2 \\ x-3 \overline{)\; x^3 - 2x^2 - 5x + 6} \\ \overset{-}{x^3} \overset{+}{-} 3x^2 \\ \hline + x^2 - 5x + 6 \\ \overset{-}{x^2} \overset{+}{-} 3x \\ \hline -2x + 6 \\ \overset{+}{-} 2x \overset{-}{+} 6 \\ \hline 0 + 0 \end{array}$$

Example 10	$\int \frac{-2}{3} \sin x \; dx$

Solution

$$I = \int \frac{-2}{3} \sin x \; dx = \frac{-2}{3}(-\cos x) + c = \frac{2}{3}\cos x + c$$

Example 11	$\int \frac{1}{\cos^2 x} dx$

Solution

$$I = \int \frac{1}{\cos^2 x} dx = \int \sec^2 x \, dx = \tan x + c$$

Example 12	$\int (5 + 5\cot^2 x) dx$

Solution

$$I = \int (5 + 5\cot^2 x) dx = \int 5(1 + \cot^2 x) dx = \int 5 \csc^2 x \, dx = -5 \cot x + c$$

Example 13	$\int \frac{\csc x \; \cos x}{\sin x} dx$

Solution

$$I = \int \frac{\csc x \; \cos x}{\sin x} dx = \int \csc x \cdot \frac{\cos x}{\sin x} dx = \int \csc x \; \cot x \, dx = -\csc x + c$$

Example 14	$\int \pi^{\frac{2}{3}} dx$

Solution

$I = \int \pi^{\frac{2}{3}} dx = \pi^{\frac{2}{3}} x + c$

Example 15	Let f be a function such that $f^{\backslash}(x) = 5$, $\forall\, x \in \mathcal{R}$ and $f(0) = -2$. Find $f(4)$

Solution

$f^{\backslash}(x) = 5$
$f(x) = 5x + c$
$f(0) = -2$
$\therefore -2 = 5(0) + c$
$c = -2$
$f(x) = 5x - 2$
$f(4) = 5(4) - 2 = 20 - 2 = 18$

Example 16 28 January 13. 2007	The graph of $y = f(x)$ intersects the line $y = x$ at $x = 0$ and $x = 1$. Find $f(x)$ If $f^{\backslash\backslash}(x) = 1 + 2x - 3x^2$

Solution

$f^{\backslash\backslash}(x) = 1 + 2x - 3x^2$
$f^{\backslash}(x) = x + x^2 - x^3 + c_1$
$f(x) = \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + c_1 x + c_2$
$f(0) = 0$
$0 = 0 + 0 - 0 + 0 + c_2$
$c_2 = 0$
$f(1) = 1$
$\therefore 1 = \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + c_1$
$\frac{6 + 4 - 3}{12} + c_1 = 1$
$c_1 = 1 - \frac{7}{12}$
$c_1 = \frac{5}{12}$
$f(x) = \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{5}{12}x$

Homework

1	14 January 6, 1996 Evaluate the following integral $\int \pi^2 dx$
2	7 June 17, 1993 Let f be a function such that $f^{\backslash}(x) = 2\ ,\ \forall x \in \mathcal{R}$ and $f(0) = 5$. Find $f(7)$
3	32 August 02, 2008 The slop of a curve $y = f(x)$ is given by $m(x) = 2x + \sin x + 1$. Find $f(x)$ knowing that this curve passing through the point $P(\ 0\ ,1\ )$
4	24 July 20th , 2000 Evaluate the following integral $\int \frac{2t - 5}{\sqrt[3]{t}} dt,$